Similarly

$$\frac{e^2 n_i v_i}{m_i} = \frac{e^2 n_e v_e}{m_i} + \frac{e}{m_i} J \quad \text{--- (11)}$$

adding (10) & (11) then we get –

$$\frac{e^2 n_e v_e}{m_e} + \frac{e^2 n_i v_i}{m_i} = e^2 \left[\frac{n_i v_i}{m_e} + \frac{n_e v_e}{m_i}\right] + eJ\left[\frac{1}{m_i} - \frac{1}{m_e}\right]$$

$$= \frac{e^2}{m_e m_i}\left(m_i n_i v_i + n_e v_e m_e\right) + J\left[\frac{e}{m_i} - \frac{e}{m_e}\right]$$

$$= \frac{e^2}{m_e m_i} v \rho_m + J\left[\frac{e}{m_i} - \frac{e}{m_e}\right]$$

[using (1) & (3)]

$$\left(\frac{e^2 n_e v_e}{m_e} + \frac{e^2 n_i v_i}{m_i}\right) \times B_0 = + \frac{e^2 \rho_m}{m_e m_i}(v \times B_0) + \left[\frac{e}{m_i} - \frac{e}{m_e}\right] J \times B_0 \quad \text{--- (12)}$$

using relations —

$$\frac{\partial J}{\partial t} + e v_i \nabla(n_i v_i) - e v_e \nabla(n_e v_e) + e n_i (v_i \cdot \nabla) v_i - e n_e (v_e \cdot \nabla) v_e$$

$$= \nabla\left(\frac{e p_e}{m_e} - \frac{e p_i}{m_i}\right) + e^2\left(\frac{n_e}{m_e} + \frac{n_i}{m_i}\right)E$$

$$- \frac{e}{c}\left(\frac{1}{m_e} - \frac{1}{m_i}\right)(J \times B_0) + \frac{e^2}{c}\frac{\rho}{m_e m_i}(v \times B_0)$$

$$+ \text{Collision term}$$

it has been observed that the collision term can be expressed as $-\nu J$ where ν is the collision frequency is given by

$$\nu = \frac{e^2}{\sigma}\left(\frac{n_e}{m_e} + \frac{n_i}{m_i}\right)$$ —

therefore, the momentum transfer equations can be expressed as :-

$$\frac{\partial J}{\partial t} + e v_i \nabla(n_i v_i) - e v_e \nabla(n_e v_e) + e n_i (v_i \cdot \nabla) v_i$$
$$- e n_e (v_e \cdot \nabla) v_e = \nabla\left(\frac{e p_e}{m_e} - \frac{e p_i}{m_i}\right) + e^2\left(\frac{n_e}{m_e} + \frac{n_i}{m_i}\right)E$$
$$- \frac{e}{c}\left(\frac{1}{m_e} - \frac{1}{m_i}\right)(J \times B_0) + \frac{e^2}{c}\left(\frac{\rho}{m_e m_i}\right)(v \times B_0)$$
$$- \frac{e^2}{\sigma}\left(\frac{n_e}{m_e} + \frac{n_i}{m_i}\right)J$$

This is known as generalized Ohm's law.

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Analysis

Let us define the mean velocity v, the current density J and the mass density ρ are —

$$v = \frac{n_e m_e v_e + m_i n_i v_i}{m_e n_e + n_i m_i} \quad \text{--- (1)}$$

$$J = e(n_i v_i + n_e v_e) \quad \text{--- (2)}$$

$$\rho = n_i m_i + n_e m_e \quad \text{--- (3)}$$

Also we consider the linearized momentum transfer eqn for electrons and ions

$$\frac{\partial v_e}{\partial t} = -\frac{e}{m_e} E - \frac{e}{m_e c}(v_e \times B_0) - \delta_1 (v_e - v_i) \quad \text{--- (4)}$$

$$\frac{\partial v_i}{\partial t} = \frac{e}{m_e} E + \frac{e}{m_i c}(v_i \times B_0) - \delta_2 (v_i - v_e) \quad \text{--- (5)}$$

Where δ_1 & δ_2 are the collision frequencies.

For ~~simplicity~~ here we assume that $n_e = n_i = n$ and $m_i \gg m_e$ and we find that

$$v = v_i + \frac{m_e}{m_i} v_e \quad \text{--- (6)}$$

$$\rho \approx m_i n_i \approx m_i n \quad \text{--- (7)}$$

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$$J = ne(v_i - v_e) \approx ne\left[v - \left(\frac{m_e}{m_i} + 1\right)v_e\right] \quad \text{—— (8)}$$

Hence

$$v_e = v - \frac{J}{ne} \quad \text{—— (9)}$$

$$v_i = v + \frac{m_e}{\rho e} J \quad \text{—— (10)}$$

$$v_e - v_i \approx -\frac{J}{ne} \quad \text{—— (11)}$$

now from eqn (4), (9) & (11) we get –

$$\frac{\partial v}{\partial t} - \frac{1}{ne}\frac{\partial J}{\partial t} = -\frac{e}{m_e}E - \frac{e}{m_e c}(v \times B_0) + \frac{\nu_1}{ne}J + \frac{1}{m_e n e}(J \times B_0) \quad \text{—— (12)}$$

from eqn (5), (10) & (11) we get

$$\frac{\partial v}{\partial t} + \frac{m_e}{\rho e}\frac{\partial J}{\partial t} = \frac{e}{m_i}E + \frac{e}{m_i c}(v \times B_0) + \frac{m_e}{\rho m_i c}(J \times B_0) - \frac{\nu_2}{ne}J \quad \text{—— (13)}$$

Substracting (12) from (13) and some adjustment we get approximately as

$$\frac{1}{ne}\frac{\partial J}{\partial t} = \frac{e}{m_e}E + \frac{e}{m_e c}(v \times B_0) - \frac{1}{m_e n e}(J \times B_0) - \frac{\nu}{ne}J \quad \text{—— (14)}$$

where $\nu = \nu_1 + \nu_2$

eqn (14) can be written as

$$J = \frac{ne^2}{m_e \nu}\left[E + \frac{1}{c}(v \times B_0)\right] - \frac{e}{m_e \nu c}(J \times B_0) - \frac{1}{\nu}\frac{\partial J}{\partial t} \quad \text{—— (15)}$$

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This is the generalised Ohm's law.

for steady state that is $\frac{\partial J}{\partial t} = 0$

therefore (15) becomes

$$J = \frac{ne^2}{m_e \gamma}\left[E + \frac{1}{c}(v \times B_0)\right] - \frac{e}{m_e \gamma_e}(J \times B_0)$$

$$\Rightarrow \frac{J}{\sigma} = \left[E + \frac{1}{c}(v \times B_0)\right] - \frac{1}{nec}(J \times B_0) \quad \text{(16)}$$

This is modified form of generalised ohm's law.

where $\sigma = \frac{ne^2}{m_e \gamma}$

the electrical conductivity in presence of constant electric field and zero magnetic field.

SOME MHD Physics :-

generalised ohm's law for MHD generator is

$$\frac{J}{\sigma} = E + \frac{1}{c}(v \times B_0) - \frac{1}{nec}(J \times B_0) \quad \text{(1)}$$

where $\sigma = \frac{ne^2}{m_e \gamma}$ the electrical conductivity for the electrons n is the number density, m_e is the mass & e is the charge per particle. Since B_0 is ⊥ to the z axis & the flow velocity ∥ to the x axis we can write.

$$J_x = \frac{\sigma}{1+\alpha^2}\left[E_x - \alpha\left(E_y - \frac{v}{c}B_0\right)\right] \quad \text{(2)} \qquad J_y = \frac{\sigma}{1+\alpha^2}\left[E_y + \alpha\left(E_x - \frac{v}{c}B_0\right)\right] \quad \text{(3)} \qquad J_z = \sigma E_z \quad \text{(4)}$$

or is known as electron gyro-frequency $\alpha = \frac{\sigma B_0}{nec} = \frac{ne^2}{m_e \gamma}\cdot\frac{B_0}{nec} = \frac{\omega}{\gamma}$

eliminating E_y between (2) & (3) we get

$$J_x = -\alpha J_y + \sigma E_x \quad \text{(6)}$$

eliminating E_x between (2) & (3) we get

$$J_y = \alpha J_x + \sigma\left(E_y - \frac{v}{c}B_0\right) \quad \text{(7)}$$

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FARADAY Type generators :-

In faraday type of MHD generators loads are connected so that the primary current flow in the gas is in the direction of $v \times B_0$. Hence no current in the direction flow, i.e $J_x = 0$ ——— (8)

and the or MHD generator β is defined as $E_y = \beta\left(\frac{v}{c} B_0\right)$ ——— (9)

using (8) eqn (7) becomes —

$$J_y = \sigma\left(E_y - \frac{v}{c} B_0\right) = \sigma\left(\frac{\beta v B_0}{c} - \frac{v}{c} B_0\right)$$

$$\Rightarrow J_y = -\sigma \frac{v B_0}{c}(1-\beta) \quad \text{——— (10)}$$

then eqn (6) becomes —

$$E_x = \frac{\alpha}{\sigma} J_y$$

$$= -\frac{\alpha}{\sigma}\, \sigma \frac{v B_0}{c}(1-\beta)$$

$$\Rightarrow E_x = -\frac{\alpha}{c} v B_0 (1-\beta) \quad \text{——— (11)}$$

To show that β, defined by eqn (9) gives the eqn of generator loading co-efficient (16) we determine in numerator and denominator —

$$(E \cdot J) = E_y J_y = -\beta \frac{v^2}{c^2} B_0^2 \sigma (1-\beta) \quad \text{——— (12)}$$

$$\frac{1}{\sigma} J^2 = \sigma \frac{v^2}{c^2} B_0^2 (1-\beta)^2 \quad \text{——— (13)}$$

$$(J \times B_0)_x = J_y B_0 = -\frac{v}{c} \sigma B_0^2 (1-\beta) \quad \text{——— (14)}$$

eqn (12), (14) verify eqn (9) Also it is seen that,

$$\boxed{\frac{v}{c}(J \times B_0)_x + \frac{1}{\sigma} J^2 = (E \cdot J)} \quad \text{——— (15)}$$

Hence, the sum of workdone by the gas against the field and internal dissipation equals the output.

Sound waves

As an introduction to ion waves, let us briefly review the theory of sound waves in ordinary air. Neglecting viscosity, we can write the Navier-Stokes eqn, which describes these waves, as

$$\rho\left[\frac{\partial v}{\partial t}+(v\cdot\nabla)v\right]=-\nabla p=-\frac{\gamma p}{\rho}\nabla\rho \quad \text{(1)}$$

The equation of continuity is

$$\frac{\partial\rho}{\partial t}+\nabla(\rho v)=0 \quad \text{(2)}$$

Linearizing about a stationary equilibrium with uniform p_0 and ρ_0, we have

$$-i\omega\rho_0 v_1=-\frac{\gamma p_0}{\rho_0}ik\rho_1 \quad \text{(3)}$$

$$-i\omega\rho_1+\rho_0 ikv_1=0 \quad \text{(4)}$$

where we have again taken a wave dependence of the form $\exp[i(\vec{k}\cdot\vec{r}-\omega t)]$. For a plane wave with $\vec{k}=k\hat{x}$ and $\vec{v}=v\hat{x}$, we find, eliminating ρ_1,

$$-i\omega\rho_0 v_1=-\frac{\gamma p_0}{\rho_0}ik\frac{\rho_0 ikv_1}{i\omega}$$

$$\omega^2 v_1=k^2\frac{\gamma p_0}{\rho_0}v_1$$

$$\Rightarrow \frac{\omega}{k}=\left(\frac{\gamma p_0}{\rho_0}\right)^{\frac{1}{2}}=\left(\frac{\gamma k_B T}{M}\right)^{\frac{1}{2}}=c_s \quad \text{(5)}$$

This is the expression for the velocity c_s of sound waves in a neutral gas. (The waves are p   are waves propagating from one layer to the next by collisions among the air molecules). In a plasma with no neutrals and few collisions, an analogous phenomena occurs. This is called an <u>ion-acoustic</u> wave or simply, an <u>ion wave</u>.

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